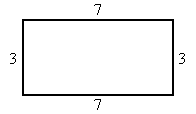
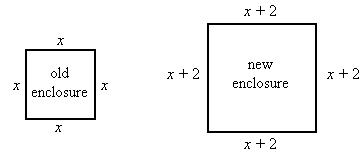
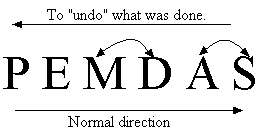
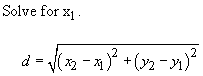
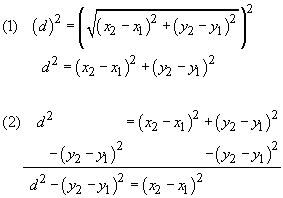
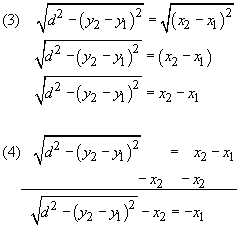
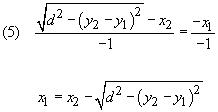
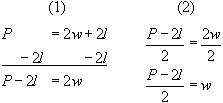
Study Guide  
  
8th Grade Algebra Review  
05/18/2016

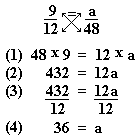
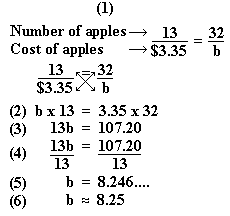
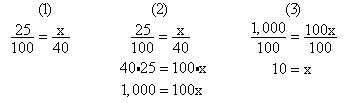
**Functions: Linear**This skill requires students to solve real world problems involving two linear functions. A linear function is a function whose graph forms a non-vertical straight line.

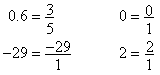
Story problems are often very difficult for students to master. It may be beneficial first to confirm that the student is comfortable with solving two linear equations outside of a real world context. To do this, students must review how to solve linear equations. In some variable equations, students are given the value of one variable. Letters expressing unknown values in equations are called variables. To find the value of the second variable, substitution must be used.  
  
If *y* = 4*x*, and *x* = 9, substitute the given value of *x* in the equation, *y* = 4*x*. The result is *y* = 4(9). Calculate the right side of the equation to get *y* = 36. Now the values of both variables are known: *x* = 9 and *y* = 36.  
  
To check the answer, set up the equation *y* = 4*x* as if *x* was an unknown variable: 36 = 4*x*. Divide each side of the equation by 4. The result is *x* = 9, so the values for *x* and *y* are both correct.  
  
Here is another situation where the value of a variable is substituted into an equation to solve for *x*.  
  
**Example 1:**  
  
Solve for *x*.  
  
 *x* = 3*y* + 9  
 *y* = - 2  
  
Solution: Substitute - 2 in place of *y* in the first equation. Then, solve for *x*.  
 *x* = 3(- 2) + 9  
 *x* = - 6 + 9  
 *x* = 3  
  
**Answer:** *x* = 3  
  
This concept can be used to solve two equations that include the same two variables.  
  
**Example 2:**  
  
Solve for *x* and *y* in the following equations.  
 *y = 2x* - 3  
 3*x = y* + 10  
  
Solution:  
If *y* = 2*x* - 3 and 3*x* = *y* + 10, there is not a number to substitute in for either of the variables. So, an expression must be substituted for a variable. In this case, the *y* variable is already isolated. It is equal to 2*x* - 3. This can be substituted in for *y* in the second equation.  
  
  
 (1) 3*x* = *y* + 10  
 3*x* = (2*x* - 3) + 10  
 (2) 3*x* = 2*x* + (- 3 + 10)  
 3*x* = 2*x* + 7  
 (3) 3*x* = 2*x* + 7  
 -2*x* -2*x*  
 *x* = 7  
 (4) *y* = 2*x* - 3  
 *y* = 2(7) - 3  
 (5) *y* = 14 - 3  
 *y* = 11  
Step 1: Substitute 2*x* - 3 for *y* in the second equation.  
Step 2: Simplify the equation by removing the parentheses and combining the like terms - 3 and 10.  
Step 3: Isolate the variable by subtracting 2*x* from each side and combining like terms.  
Step 4: Since there is now a value for one of the variables, it can be substituted back into either of the original equations. In this example, the first equation was used.  
Step 5: Calculate the right side of the equation to get *y* = 11.  
  
**Answer:** *x* = 7 and *y* = 11  
  
Several questions from this skill relate to measurement concepts. The formula for perimeter is occasionally needed to find solutions to story problems. Perimeter is the measurement around a figure.  
  
**Example 3:**  
  
Find the perimeter for the figure below.  
  
   
  
Solution:To calculate the perimeter of a figure, add the lengths of all the sides of the figure. For example, this figure has four sides measuring 3 inches, 7 inches, 3 inches, and 7 inches.   
  
 P = 7 + 3 + 7 + 3 = 20  
  
**Answer:** The perimeter of this figure is 20 inches.  
  
Story problems that involve perimeter and two linear equations can be solved by writing two equations, then using the substitution method that was reviewed earlier.  
  
**Example 4**:  
Emilio built a square sandbox for his dog with a perimeter of 15 m. Since his dog has grown, he wants to rebuild the sandbox so that each side is 2 m longer than the original sandbox. Before he buys the materials to build the new sandbox, he needs to know the perimeter. What is the perimeter of the new sandbox?  
  
Solution:  
To solve this problem it is probably best to start with drawing a picture. Because we do not know the length of the sides of the old enclosure, they can be assigned a variable, *x*. The new enclosure must have sides that are 2 m longer than the original, so the sides can be written as *x* + 2.  
   
  
(1) For the old enclosure, *x* + *x* + *x* + *x* = 15, or 4*x* = 15  
 For the new enclosure, (*x* + 2) + (*x* + 2) + (*x* + 2) + (*x* + 2) = *y*, or 4(*x* + 2) = *y*.  
(2) 4*x* = 15, so *x* = 3.75  
(3) 4(3.75 + 2) = *y*  
(4) *y* = 23  
  
Step 1: An equation can be written for the perimeter of the new and old enclosures based on the information in the problem. For the new enclosure, we do not know the perimeter so we give it a different variable, *y*.  
Step 2: The *x* must be isolated in the first equation so that it can be substituted into the second.  
Step 3: This value can be placed in the second equation for *x*.  
Step 4: Calculate.  
The perimeter of the new enclosure will be 23 m.  
  
**Answer**: 23 m  
  
Another common application of finding the solution to two equations in solving real world problems involves money.  
  
**Example 5**: Ariel wants to go snowboarding this weekend. Ice Mountain charges $24 for a lift ticket and $8 per hour of snowboarding. Flurry Ridge charges $29 for a lift ticket and $6 per hour of snowboarding. Ariel must get a lift ticket to snowboard and she wants to snowboard for 5 hours. Which ski area would be cheaper for Ariel's snowboarding trip?  
  
(1) For Ice Mountain, the cost = 24 + 8*h*.  
 For Flurry Ridge, the cost = 29 + 6*h*.  
(2) For Ice Mountain, 24 + 8(5) = 64  
 For Flurry Ridge, 29 + 6(5) = 59.  
  
Step 1: Set up two equations, one for each snowboarding location. We can use the variable *h* for the number of hours.  
Step 2: Since Ariel wants to snowboard for 5 hours, we can substitute 5 in for *h* in each of the equations and solve.  
  
**Answer**: Ice Mountain will cost her $64 and Flurry Ridge costs her only $59, so Flurry Ridge is cheaper.  
  
To help students understand the application of linear equations in solving real world problems, students can find the costs per unit of time or distance of different services with the help of a telephone book, the newspaper, or the internet. If specific cost information is not listed, call the company for information. For example, a car rental company may charge a base fee of $35 per day and $0.59 per mile or a catering company may charge a rental fee of $100 for materials and then $60 per hour for waitstaff services. Then, have students calculate which would be the cheapest and which would be the most expensive for a specified distance a rental car may drive or amount of time a catered party may last.

**Literal Equations**This study guide will focus on finding solutions to literal equations. Students will be given an equation, usually a formula, and be asked to solve the equation for a particular variable. This skill is especially useful when students need to find missing values in science experiments and technical fields.

Literal equations are comprised of literals or multiple letters, also called variables. These literals can represent known constants or numbers, or be extra variables, letters that represent a range of values to be determined. So a literal equation uses a bunch of letters to represent a specific relation. They are usually referred to as formulas.  
  
It may be beneficial to confirm that the student is comfortable with solving linear equations, equations containing one or two variables that do not contain exponents, before beginning to solve literal equations. A quick review is provided below.  
  
**Solving Linear Equations**  
  
To solve a linear equation, it is necessary to "undo" what was done to the variable in question. Another way to think about this is to do the order of operations in reverse.  
  
   
P = Parenthesis, E = Exponents, M = Multiplication, D = Division, A = Addition, S = Subtraction.  
\*\*In the normal direction, evaluate multiplication and division from left to right (whichever comes first). Evaluate addition and subtraction from left to right (whichever comes first).  
\*\*To "undo" what was done, reverse the direction.  
 •Addition "undoes" subtraction.  
 •Subtraction "undoes" addition.  
 •Multiplication "undoes" division.  
 •Division "undoes" multiplication.  
  
**Example 1:**  
 Solve for *x*.  
 *x* + 5 = 7  
    
Step 1: It is necessary to get *x* by itself on one side of the equation and everything else on the other side. Undo the "+ 5" by subtracting 5 from both sides of the equation.  
Step 2: *x* is now isolated and the other side of the equal sign is simplified.  
  
**Answer:** *x* = 2  
  
**Solving Literal Equations**  
  
The methods used for solving literal equations are the same as those used in solving linear equations, except fewer simplifications can be made since the variables cannot be combined. The goal is to solve the formula for a given variable in terms of the other variables.  
  
**Example 2:**  
  
   
   
   
Step 1: Square both sides to eliminate the radical.  
Step 3: Take the square root of both sides to eliminate the squared term on the side of the equation with the *x*-terms.  
Step 5: Divide both sides by -1 to eliminate the negative.  
  
   
**Example 3:** Solve for *w*.  
 *P* = 2*w* + 2*l*.  
    
Step 1: To isolate the *w*, subtract 2*l* from both sides of the equation to get *P* - 2*l* = 2*w*.  
Step 2: Then divide both sides of the equation by 2.  
  
   
A good way to practice this skill is to look up formulas used in different fields (physics, electronics, chemistry, etc.) and practice solving each one for a different variable. Some formulas may be more complicated than others, so try to stick to those that have the four basic operations, addition, subtraction, multiplication, and division.

**Ratio/Proportion - C**A ratio is a comparison of two numbers expressed as a quotient. Ratios can be written in three ways: a fraction (3/5), a ratio (3:5), or a phrase (3 to 5). Like fractions, ratios refer to a specific comparison. The ratios 3/5, 3:5, and 3 to 5 (as in "the ratio of cellos to violins was 3 to 5") all express the same ratio or comparison. A proportion reflects the equivalency of two ratios. The ratio 3/5 expresses the same proportion as the ratio 15/25.

To understand how ratios operate, students need to understand equivalent fractions. Fractions represent portions or parts. For every fraction, there is a corresponding portion. The fraction 1/2 communicates one portion out of two, but this specific portion can also be communicated by the fractions 2/4, 3/6, 8/16, 10/20, etc. All of these fractions are equal to 1/2 because the relationship between the numerator and denominator in 1/2 is the same relationship between the numerators and denominators in 2/4, 3/6, 8/16, and 10/20. Ratios and proportions operate in a similar manner. The ratio 2:5 communicates a specific portion. The ratio 4:10 communicates the same portion because 4:10 reduces to 2:5.  
  
**Example 1:** Sandra has 15 lollipops and 25 jellybeans. What is the ratio of lollipops to jellybeans?  
  
Answer: There are 15 lollipops to 25 jellybeans, so the ratio of lollipops to jellybeans is 15:25.  
  
Proportions occur when two ratios are equal. In a proportion the cross products of the terms are equal.  
**Example 2:** Is the following proportion True or False?  
 1/3 = 3/9  
  
   
The cross products are both equal to 9, so the proportion is TRUE. If the cross products were not equal, the proportion would be FALSE.  
  
Sometimes you must find the value of a variable in a proportion. To solve the proportion, you must find the value of the variable that makes both ratios equal.  
  
**Example 3:** What is the value of a?  
 9/12 = a/48  
  
  
  
Step 1: Find the cross products. Multiply 48 by 9 and 12 by 'a'.  
Step 2: 48 x 9 = 432 and 12 x a = 12a. Rewrite the equation with the new products.  
Step 3: Divide each side of the equation by 12 to isolate the variable 'a'.  
Step 4: Divide 432 by 12 to get a = 36.  
  
Answer: a = 36  
  
**Example 4:** Apples are 13 for $3.35. What will 32 apples cost? (Round answer to the nearest hundredth).  
  
  
  
Step 1: Write the appropriate proportion. Let b represent the cost of 32 apples.  
Step 2: Write the cross products. Multiply b by 13 and multiply 3.35 by 32.  
Step 3: Rewrite the equation with the new values.  
Step 4: Divide each side of the equation by 13 to isolate the b.  
Step 5: 107.20 ÷ 13 = 8.246153846 We only need to write 3 decimal places because we are going to round to the hundredths place (the second decimal place). The three dots above represent that the decimal number continues on past that point.  
Step 6: Round 8.246 to the hundredth place. The 6 tells us to round the 4 up to a 5. 8.246 approximately equals 8.25.  
  
Answer: $8.25  
  
Take a look at the solution. Does it make sense that 32 apples would cost $8.25? We know that 32 apples would cost more than twice as much as 13 apples (13 x 2 = 26), but less that three times as much (13 x 3 = 39). The solution is more than twice as much as $3.35 (2 x $3.35 = $6.70), but less than three times as much (3 x $3.35 = 10.05), so our answer makes sense.  
  
**Example 5**: Rachel scored 25% of the team points. If the team scored 40 points, how many did Rachel score?  
  
   
Step 1: Write a proportion to represent the problem.  
Step 2: Cross multiply. This involves multiplying the bottom of one ratio by the top of the other and vice versa. Perform the multiplication.  
Step 3: Divide both sides of the equation by 100.  
  
Answer: 10 points

**Multiplication/Division Rational No.**The following numbers are rational numbers because they can all be written as fractions.  
  
 

Multiplying and dividing rational numbers includes the calculation of whole numbers, fractions, decimals, and integers. To understand how to multiply and divide rational numbers, the student needs to know the following rules:   
  
1. A positive rational number multiplied by a positive rational number equals a positive rational number.  
 (+0.5) x (+0.2) = +0.1  
  
2. A negative rational number multiplied by a positive rational number equals a negative rational number. A positive rational number multiplied by a negative rational number equals a negative rational number.  
 (-0.5) x (+0.2) = -0.1  
  
3. A negative rational number multiplied by a negative rational number equals a positive rational number.  
 (-0.5) x (-0.2) = +0.1  
4. A positive rational number divided by a positive rational number equals a positive rational number.  
  
 (+0.1) ÷ (+0.2) = +0.5  
  
5. A negative rational number divided by a positive rational number equals a negative rational number. A positive rational number divided by a negative rational number equals a negative rational number.  
  
 (-0.1) ÷ (+0.2) = -0.5  
  
6. A negative rational number divided by a negative rational number equals a positive rational number.  
  
 (-0.1) ÷ (-0.2) = +0.5

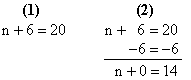
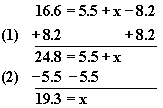
**Expressions: Subtraction**Expressions look like equations except expressions do not have equal (=) signs. Students don't "solve" an expression, they "evaluate" or "simplify" it.

The following are step-by-step examples of how to evaluate expressions.  
  
**Example 1:** Evaluate the expression for n = -2.  
 n - 6  
  
Solution: Substitute -2 in place of n and simplify.  
 -2 - 6 = -8  
  
Answer: -8  
  
**Example 2:** Evaluate the expression for x = 10.  
 35 - x  
  
Solution: Substitute 10 in place of x and simplify.  
 35 - 10 = 25  
  
Answer: 25  
  
**Example 3:** What mathematical expression best represents the word expression "a number decreased by 12"?  
  
Solution: We can represent a number with any variable, we'll use t. The number is decreased by 12, so we subtract.  
  
Answer: t - 12

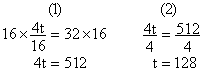
**Expressions: Division**Expressions are number sentences which do not have equal signs, but need to be evaluated or simplified.  
 Example: y - 6

**Example 1:** Evaluate the following expression when n = -3.  
   
 (1) 2/(-3)  
 (2) -2/3  
  
Step 1: Substitute the value of n into the expression.  
Step 2: Leave the expression in fraction form and place the negative sign in front.  
  
Answer: For the value of n = -3, the expression is equal to -2/3.  
  
**Example 2:** Evaluate the expression for n = -3.  
   
 (1) (-3 - 7)/2  
 (2) -10/2  
 (3) -5  
  
Step 1: Substitute the value of n into the expression.  
Step 2: Simplify -3 - 7 to get -10.  
Step 3: Divide -10 by 2 to get -5.  
  
Answer: For the value of n = -3, the expression is equal to -5.  
  
**Example 3:** Which mathematical expression best represents the word expression?  
  
 The amount of wheat, w, divided by 6 people.  
  
Use the variable, w, and show division by 6.  
  
Answer: w/6 or w ÷ 6

**Equations: Addition/Subtraction**Equations are number sentences which contain equal signs.  
 Example: n + 5 = 9  
  
Expressions are number sentences which do not have equal signs, but need to be evaluated or simplified.  
 Example: y - 6  
  
Variables are letters or symbols that represent numbers that are unknown.

The following are examples of how to solve equations.  
  
**Example 1:** Solve: n + 6 = 20  
  
  
Step 1: Write the equation.  
Step 2: Subtract 6 from both sides of the equation.  
  
Answer: n = 14  
  
**Example 2:** Solve for x.  
 16.6 = 5.5 + x - 8.2  
  
   
Step 1: Begin to isolate the variable on one side of the equation by adding 8.2 to both sides.  
Step 2: Subtract 5.5 from both sides to completely isolate the variable.  
  
Answer: x = 19.3

**Equations: Multiplication/Division**Equations are number sentences which contain equal signs:  
 Example: n + 5 = 9  
  
Expressions are number sentences which do not have equal signs, but need to be evaluated or simplified:  
 Example: y - 6

A common mistake among students first learning to multiply and divide equations is the failure to use inverse operations. For instance, in the equation 3x = 8, the student might multiply 3 to both sides of the equal sign. The correct procedure is to use the inverse operation and divide 3 from both sides (x = 8/3). By applying the inverse operation, the student can isolate the x.  
  
**Example 1:** Solve: 5n = 75  
  
   
Step 1: In the equation, solve for the value of n.  
Step 2: 5n is the same as 5 x n. Divide both sides of the equation by 5.  
Step 3: 75 ÷ 5 = 15  
  
Answer: n = 15  
  
**Example 2:** Solve for t.  
  
    
Step 1: Multiply both sides of the equation by 16 to eliminate the fraction.  
Step 2: Divide both sides if the equation by 4 to isolate the variable t.  
  
Answer: t = 128.

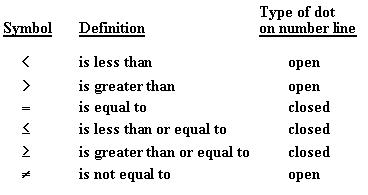
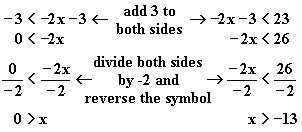
**Equations: Writing**Writing equations demonstrates the ability to understand and apply equations in real world situations. To write equations, students must interpret data presented in word problems and formulate numerical equations.  
  
Equations are number sentences which contain equal signs.  
 Example: n + 5 = 9  
  
Expressions are number sentences which do not have equal signs, but need to be evaluated or simplified.  
 Example: y - 6

The following is a step-by-step example.  
  
**Example:** Today, Jim worked 2 minutes longer than he did yesterday. Yesterday, he worked for 19 minutes. How long did Jim work today?  
  
 (1) yesterday (y) = 19 and today (t) = y + 2  
 (2) t = y + 2  
 (3) t = 19 + 2  
  
Step 1: Identify the known amounts.  
Step 2: Develop an equation from the known amounts.  
Step 3: Using substitution, insert the known amounts and solve.  
  
Answer: t = 21 minutes

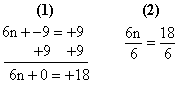
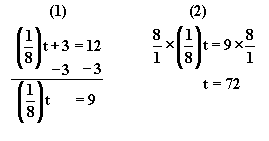
**Story Problems**Some story problems present equation and expression problems in text format.  
  
Equations are number sentences which contain equal signs.  
 Example: n + 5 = 9  
  
Expressions are number sentences which do not have equal signs, but need to be evaluated or simplified.  
 Example: y - 6

**Example:** Avital ran 10 miles more than she did yesterday. Yesterday, she ran 9 miles. How many miles did Avital run today?  
  
  
 (1) yesterday (y) = 9 and today (t) = y + 10  
 (2) t = y + 10  
 (3) t = 9 + 10 = 19  
  
Step 1: Identify the known amounts.  
Step 2: Develop an equation from the known amounts.  
Step 3: Using substitution, insert the known amounts and solve.  
  
Answer: Avital ran 19 miles today.

**Inequalities - B**An inequality is a number sentence that uses "is greater than", "is less than", or "is not equal to" symbols. For example, 6n > 4 is a number sentence with an inequality symbol.

It may be useful to review the inequality symbols.  
  
   
**Example 1:** Solve for y.  
  
 8y > 40  
  
   
Get the variable being solved for (y) on one side of the inequality and the whole number on the other. To do this, divide both sides by 8.  
  
The correct answer is that y is greater than 5.  
  
Inequalities can be represented as a value on a number line. The following number line represents the inequality   
   
  
**Example 2:** Which inequality represents the value shown on the number line below?  
  
   
 A. n < 3  
 B. n > 3  
 C. n = 3  
  
The answer is A. n < 3 because the dot on the number line is open.  
  
**Example 3:** Solve the following inequality.  
 -3 < -2x - 3 < 23  
  
 Step 1: We can solve by separating the inequality into two inequalities as shown below.  
 -3 < -2x - 3 and -2x - 3 < 23  
  
Step 2: Solve both inequalities.  
  
  
  
The answer is -13 < x < 0.  
  
NOTE: -13 < x < 0 is the same as 0 > x > -13.

**Equations: Two-Step**Two-step equations require students to perform two operations before solving the equation.  
  
For example, the equation 3x - 3 = 18 requires adding 3 to both sides of the equation and dividing both sides of the equation by 3.

**Example 1:** Find the value of n in the equation 6n - 9 = 9  
  
   
Step 1: In the equation, solve for the value of n by first getting n alone. Add +9 to both sides of the equation.  
Step 2: Divide both sides of the equation by 6.  
  
Answer: n = 3  
  
**Example 2:** Solve for t.  
 (1/8)t + 3 = 12  
  
   
Step 1: To get the variable alone, subtract 3 from both sides of the equation.  
Step 2: Isolate the 't' by multiplying by the reciprocal of 1/8, which is 8/1.  
  
Answer: t = 72

**Solving Equations: Substitution**Letters expressing unknown values in equations are called variables. In these two variable equations, students are given the value of one variable. To find the value of the second variable, substitution must be used.

If y = 4x, and x = 9, substitute the given value of x, in the equation, y = 4x. The result is y = 4(9). Calculate the right side of the equation to get y = 36. Now the values of both variables are known: x = 9 and y = 36.  
  
To check the answer, set up the equation y = 4x as if x was an unknown variable: 36 = 4x. Divide each side of the equation by 4. The result is x = 9, so the values for x and y are both correct.  
  
**Example 1:** Solve for x.  
 x = 3y + 9  
 y = -2  
  
Solution: Substitute -2 in place of 'y' in the equation. Then, solve for x.  
 x = 3(-2) + 9  
 x = -6 + 9  
 x = 3  
  
When we are not given enough information, the problem cannot be solved.  
  
**Example 2:** Solve for t.  
 t = s -12  
 r = s + 3  
  
Solution: Since we do not know the value of either 's' or 'r', we cannot determine the value of t.

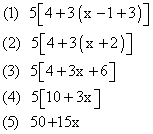
**Missing Elements - E**Equations will be presented with either a number or operational symbol omitted. Students must solve the equation by determining the missing part.

It may be useful to review the following examples with the student. Help him or her determine whether the missing element is an operational symbol, number, or expression. When the nature of the missing element is determined, make a list of possible answers. When you have the list completed, test each element to determine the correct solution.  
  
  
**Example 1:** Find the missing symbol.  
 13 ? 14 = -1  
  
Answer: The missing element is an operational symbol. Possible answers, therefore, are symbols for addition, subtraction, multiplication, or division. The answer is -1, so we can rule out addition and multiplication immediately. The subtraction symbol fits because 13 - 14 = -1.  
  
**Example 2:** What is the value of N given that A = 87?  
  
 10 x N = A - 7  
  
 (1) 10 x N = 87 - 7  
 (2) 10 x N = 80  
 (3) 10 ÷ 10 x N = 80 ÷ 10  
 (4) N = 8  
  
Step 1: Substitute the value of A into the equation.  
Step 2: Calculate 87 - 7.  
Step 3: Divide each side of the equation by 10 to isolate the missing element on one side of the equation.  
Step 4: The value of N is 8.

**Rates**Rates refer to the price per unit of a particular commodity. Pricing tactics often cause confusion for today's consumer. Products are sold at 3 for one amount or 5 for another. Calculating rates allows students to figure out the exact cost of the items.

**Example:** Apples are 5 for $1.25 and bananas are 6 for $1.80, which fruit is less expensive if you only buy one?  
  
 (1) $1.25 ÷ 5 = $0.25 --> cost of one apple  
 (2) $1.80 ÷ 6 = $0.30 --> cost of one banana  
 (3) $0.25 < $0.30  
  
Step 1: Determine the cost of one apple by dividing 1.25 by 5. One apple costs $0.25.  
Step 2: Determine the cost of one banana by dividing 1.80 by 6. One banana costs $0.30.  
Step 3: Compare the cost of the two pieces of fruit. $0.25 is less than $0.30.  
  
Answer: One apple is less expensive than one banana.

**Properties - E**Mathematical properties are, in essence, statements about the ways in which certain combinations of numbers relate to each other. The student should be familiar with the following properties of multiplication and addition: multiplication by 0, associative, reciprocal, commutative, and order of operations.

Please review the following rules with the student:  
  
1. Multiplication by 0: the product of any integer and 0 equals 0.  
  
 -3 x 0 = 0 3 x 0 = 0  
  
2. Associative Property of Addition: (a + b) + c = a + (b + c).  
  
 (1 + 2) + 3 = 1 + (2 + 3)  
  
3. Associative Property of Multiplication: (a x b) x c = a x (b x c).  
  
 (1 x 2) x 3 = 1 x (2 x 3)  
  
4. Reciprocals: two numbers are reciprocals if their product equals 1.  
  
  
5. Commutative Property of Addition: a + b = b + a  
  
 1 + 2 = 2 + 1  
  
6. Commutative Property of Multiplication: a x b = b x a  
  
 1 x 2 = 2 x 1  
  
7. Order of Operations:  
A. When calculations for a given expression or equation require both addition and multiplication, the rule is to multiply first and add second.  
  
 (3)(2) + 3 = ?  
 6 + 3 = ?  
 6 + 3 = 9  
  
B. The number outside the parentheses is multiplied with each number within the parentheses:  
x(y + z) = xy + xz.  
  
 (1) 3(x + y)  
 (2) 3(x) + 3(y)  
 (3) 3x + 3y  
  
C. If a given expression contains both parentheses and brackets, calculations should be completed working from the innermost parentheses or bracket outward.  
  
   
The following are sample questions using the above properties.  
  
**Example 1:** Which answer best completes the number sentence?  
 5 = (5 x 4) + (5 x 6)  
  
 A. x (4 + 6)  
 B. + (4 + 6)  
 C. x (20 + 30)  
 D. + (4 x 6)  
  
Answer: A (because of rule 7A)  
  
**Example 2:** Which answer best completes this number sentence?  
 0.32 + 0.45 + -62 = ?  
  
 A. -0.15  
 B. 0.32  
 C. 0.15  
 D. 0.45  
  
Answer: C  
  
**Example 3:** Which one of the following best completes the number sentence?  
 (2.3 + 3.1) x 5.6 = ?   
  
 A. 3.5 x 5.6  
 B. 7.2 x 5.6  
 C. 9.1 x 5.6  
 D. 5.4 x 5.6  
  
Answer: D (because of rule 7B)  
  
**Example 4:** What is the value of n in the following statement?  
 13 x (3.4 x 0) = n  
  
 A. 44.2  
 B. 0  
 C. 13  
 D. 3.4  
  
Answer: B (because of rule 1)

**Multiple-step Story Problems - F**These problems are designed to assess a student's ability to interpret data from word problems (some contain both relevant and irrelevant data) and develop solutions which require two or more operations (or steps) to solve.

It may be helpful to develop a series of multiple step word problems that relate to the student's activities, such as allowance. The following is a step-by-step example of a multiple-step story problem.  
  
**Example 1:** Shinika bought a dozen eggs and 3 gallons of milk at the grocery store. If each gallon of milk was $2.60 and the total bill was $10.25, how much is one egg?  
  
 (1) $10.25 - 3($2.60) = ?  
 (2) $10.25 - $7.80 = ?  
 (3) $10.25 - $7.80 = $2.45  
 (4) $2.45 ÷ 12 = ?  
 (5) $2.45 ÷ 12 = $0.204166...  
 (6) $0.204166 ~ $0.20  
  
Step 1: The cost of the dozen eggs that Shinika bought can be figured by subtracting the cost of the milk from the total bill. Remember that Shinika bought 3 gallons of milk, so multiply $2.60 by 3 and subtract that product from 10.25.  
Step 2: $2.60 times 3 equals $7.80.  
Step 3: Subtract $7.80 from $10.25 to get the cost of the dozen eggs.  
Step 4: The cost of the dozen eggs is $2.45. There are 12 eggs in one dozen, so divide $2.45 by 12 to determine the cost of one egg.  
Step 5: One egg costs $0.204166... Round this number to the nearest cent.  
  
Answer: One egg costs approximately $0.20.

**Example 2:** Michael, Alex, Joe, and Jesse were all comparing their CD collections. Michael had 1/5 less than Jesse. Before he bought 10 new CDs, Joe had 5 times as many as Jesse. Alex had twice as many as Joe. Joe has 110 CDs. How many CDs did Michael have?  
  
 (1) 110 - 10 = 100  
 (2) 100 ÷ 5 = 20  
 (3) 20 ÷ 5 = 4  
  
Step 1: Determine the number of CDs that Joe had before he bought the 10 new ones.  
Step 2: Before Joe bought the 10 new CDs, he had 5 times as many as Jesse, so divide 100 by 5 to determine the number of CDs that Jesse had.  
Step 3: We know that Michael has 1/5 as many CDs and Jesse, so divide 20 by 5 to determine the number of CDs that Michael has.  
  
Answer: 4 CDs  
  
Irrelevant information: Alex had twice as many as Joe.

**Irrelevant/Missing Info - B**Students are presented with problems which assess their ability to read a word problem and use only pertinent information to determine the solution.

The following is an example of a story problem that contains irrelevant information.  
  
**Example:** Cameron is twice as old as Gerardo. Chris is half Gerardo's age. Jelena, Cameron's 25 year old sister, is 7 years older than Cameron. Based on this information, how old is Gerardo?  
  
Solution: Make a list of the ages.   
  
 1. Cameron's age = 2 x Gerardo's age  
 2. Chris's age = 1/2 x Gerarado's age  
 3. Jelena's age = Cameron's age + 7

The question requires Gerardo's age to be found. The only age we are given is Jelena's, 25. We can determine that Cameron is 18 because we know that he is 7 years younger than Jelena. Because we know that Cameron is 18, we know that Gerardo is 9 since 9 is half of 18. Therefore, Gerardo is 9 years old. Chris's age was the irrelevant piece of information.  
  
Many word problems contain irrelevant information. Help the student practice breaking down information in story problems in his or her math textbook. As he or she learns this skill, he or she will become better at solving word problems.