Study Guide

7th Grade Number Sense Review
05/18/2016

**Divide By Decimals: Story Problems**In a division problem, the dividend is the number that is to be divided. The divisor is the number that is divided into the dividend. The quotient is the answer to the division problem. In this skill, students are asked to divide a whole number by a decimal number within the context of a real world problem.

**Example 1:** Divide. Round your answer to the nearest tenth, if necessary.
 
 
 
Step 1: When dividing any number by a decimal, the decimal point in the divisor must be moved to the right the same number of spaces as there are decimal places (so that it becomes a whole number). The decimal point must be moved the same number of places in the divisor and dividend. In this case, move the decimal point one place to the right in the divisor and on place to the right in the dividend.
Step 2: Write the new division problem, making sure to write the decimal point straight above its current location, and begin the division process. Since 1 and 12 cannot be divided by 68, determine the number of times that 68 will go into 120 (1 time, with 52 left over). Write the 1 above the 0 in 120, and subtract 68 from 120.
Step 3: Bring the next zero straight down to turn the 52 into 520. The number 68 will go into 520 seven times with 44 left over. Write the 7 to the right of the 1c6, and subtract 476 (68  7 = 476) from 520.
Step 4: Add another zero after the decimal point under the division bar and bring it straight down to turn the 44 into 440. The number 68 will go into 440 six times with 32 left over. Write the 6 beside the decimal point, and subtract 408 (68  6 = 408) from 440.
Step 5: The problem stated to round the answer to the tenths place, so the division must be taken out to the hundredths place in order to determine how to round the number. Add another zero after the last zero under the division bar, then bring it straight down to turn the 32 into 320. The number 68 will go into 320 four times with 48 left over. Write the 4 next to the 6 and subtract 272 (68  4 = 272) from 320.
Step 6: Round 17.64 to the tenths place. The 4 in the hundredths place makes the 6 stay the same, so 17.64 rounded to the tenths place would be 17.6. NOTE: If the 4 had been a 5 or greater, the 6 would have been rounded to a 7.

**Answer:** 17.6

Once the student is comfortable dividing whole numbers by decimal numbers, he or she will be ready to divide these numbers in the context of story problems.

**Example 2:** A quart of paint covers a 62.5 square foot area. If Jason wants to paint a wall that is 160 square feet, how many quarts of paint will he use?

 (1) 160 ÷ 62.5 = ?
 (2) 160 ÷ 62.5 = 2.56

Step 1: Read the problem to determine what operations are needed. To find the number of quarts of paint used, divide 160 square feet by 62.5 square feet.
Step 2: Perform the division and simplify.

**Answer:** 2.56 quarts

**Example 3:** A bag contains 20 cups of sugar. If Alex needs 1.75 cups of sugar to make a batch of fudge, how many batches of fudge can he make with the bag of sugar? Round your answer to the nearest whole number, if necessary.

 
Step 1: Read the problem to determine what operations are needed. To find the number of batches of fudge that can be made with 20 cups of sugar, divide 20 cups by 1.75 cups.
Step 2: Perform the division and simplify.
Step 3: Round the answer to the correct place value (nearest whole). The number 11.4 can be rounded to 11.

**Answer:** 11 batches

An activity that can help reinforce this concept is to measure out 10 cups of sugar into a bowl. Give the student a ? cup measuring utensil and explain that ? cup is equal to 0.25 cups. Next, have the student count how many scoops of sugar it takes to empty the bowl of sugar. Finally, explain that when 10 cups of sugar are divided into 0.25-cup scoops, the number of 0.25 scoops is equal to 40. Therefore, 10 ? 0.25 = 40.

**Order Numbers: Fractions/Dec./Percents**In this grade level, students will make comparisons between fractions, decimals, and percents. They will then determine the order of least to greatest or greatest to least.

It may be necessary to review fractions, decimals, and percents with the student. As the student practices comparisons, remind him or her that fractions, decimals, and percents all represent specific portions or parts. For example, the shaded portion in the diagram below can be represented by the fraction 3/4, the percent 75%, or the decimal 0.75.

 
To compare fractions, decimals, and percents, each number must be converted to the same form. For each problem, there are several conversions that can be made to compare one number type to the other. However, converting to decimals first is almost always the easiest method.

**Converting Fractions or Percents into Decimals:**

To convert a fraction into a decimal, simply divide the numerator (top number) by the denominator (bottom number). In the example 4/5, divide 4 by 5. The answer is 0.8, the decimal equivalent to 4/5. To convert a percent into a decimal, we must recall that percent means "per one hundred." Therefore, 54% is equivalent to 54/100. To convert this fraction into a decimal, divide 54 by 100. Hence, 54% = 0.54. (Dividing by 100 is the same as moving the decimal point two places to the left. Since the decimal point in 54 is to the right of the 4 (54.0), moving the decimal point two places to the left would also produce 0.54.)

**Converting Decimals or Percents into Fractions:**

When converting a decimal into a fraction, remember that decimals are parts of a whole having place values in powers of ten. For example, 3.23 expresses 3 wholes and 23 hundredths of a whole. Therefore, to change 3.23 into a fraction, rewrite it as 3 23/100, or 323/100. Reduce the fraction, if possible. When converting a percent into a fraction, write the percent over 100, and if possible, reduce the fraction. Hence, 54% = 54/100, which can be divided by their common factor of 2, yielding 27/50.
**Converting Fractions or Decimals into Percents:**

When converting a fraction into a percent, first change the fraction into a decimal by dividing the numerator by the denominator. In the example above, 4/5 became 0.8. To change a decimal into a percent, multiply it by 100, or move the decimal point two places to the right. Thus, 0.8 becomes 80%. Hence 4/5 = 80%.

**Example 1:**
Put the following numbers in order from least to greatest.
 
Step 1: Determine a good method to compare the given numbers. In most problems, including this example, it is relatively simple to convert all of the given numbers into decimals.
Step 2: Convert 4/5 into a decimal by dividing 4 by 5, yielding 0.8.
Step 3: Convert 75% into a decimal by dividing it by 100, or moving the decimal point two places to the left. 75% = 0.75.
Step 4: 0.9 is given in the form of a decimal already.
Step 5: Order these numbers from least to greatest, 0.75, 0.8, and 0.9. Therefore, the answer is 75%, 4/5, and 0.9.


**Example 2:**
Put the following numbers in order from greatest to least.

 
Step 1: Determine a good method to compare the given numbers. In most problems, including this example, it is relatively simple to convert all of the given numbers into decimals.
Step 2: Convert 270% into a decimal by recalling that percent means "per one hundred". Therefore, 270% = 270/100, which is the same as moving the decimal point two places to the left. 270% = 2.70 = 2.7.
Step 3: Convert 8/3 into a decimal by dividing 8 by 3. The result is approximately 2.6667.
Step 4: Order these numbers from greatest to least, 2.8, 2.7, 2.6667. Therefore, the answer is 2.8, 270%, and 8/3.


A good activity to reinforce this skill could involve a bag of snack food or dollar bill (broken into change). The student could be asked if they would rather have 37%, 3/8, or 0.34 of the contents of the bag or the dollar. Whatever answer they decide on is how much they get, as long as they provide reasoning for why their choice is the best choice.

**Equivalent Fractions - B**A fraction is made up of two parts: a numerator and a denominator. The numerator is the number on the top of the fraction and the denominator is the number on the bottom of the fraction. For example, in the fraction 4/5, 4 is the numerator and 5 is the denominator. A mixed number is a combination of a whole number and a fraction. An improper fraction is a fraction in which the numerator is larger than or equal to the denominator. An improper fraction can be rewritten as a mixed number or as a whole number.

**Example 1:** Find the missing number.

 
Solution: Convert 4 2/3 into an improper fraction by multiplying the whole number by the denominator (4 x 3 = 12), then adding that product to the numerator (12 + 2 = 14). 4 2/3 can be written as 14/3.

The missing number is 14.

**Example 2:** Find the missing number.
  
Step 1: Replace the question mark with the variable, 'N'. Then cross multiply to begin the process of determining the missing number. Multiply the denominator of the first fraction by the numerator of the second fraction (N x 36 = 36N). Next multiply the denominator of the second fraction by the numerator of the first fraction (12 x 12 = 144).
Step 2: Place an equal sign between the two products. Divide both sides of the equation by 36.
144 ÷ 36 = 4

Answer: N = 4

**Equivalent Forms: Dec./Fract./Percent**Fractions can be written as decimals and percents. For example, 1/4 is 0.25 or 25%. The numerator of a fraction is the number on the top of the fraction and the denominator of a fraction is the number on the bottom of the fraction.

Develop a series of fractions and decimals and help the student find the equivalent forms. The table below will help get you started.

 
**Example 1:** Write 2/5 as a decimal and as a percent.

 
Step 1: Every fraction can also be written as a division problem by dividing the numerator by the denominator.
Step 2: Complete the division problem to write 2/5 as a decimal.
Step 3: To write a decimal as a percent, multiply the decimal number by 100. This involves moving the decimal point two places to the right.

Answers: Decimal 0.4 and Percent 40%

**Example 2:** Write 8.2% as a decimal and as a fraction.

 
Step 1: To change a percent into a decimal, divide the percent by 100. This involves moving the decimal point two places to the left.
Step 2: The decimal 0.082 is read "eighty-two thousandths," so it can be written as the fraction 82/1000.
Step 3: Since 82 and 1000 can both be divided by 2, the fraction can be reduced to 41/500.

Answers: Decimal 0.082 and Fraction 41/500.

**Divisibility/Multiples/Factors - B**Divisibility occurs when one number is divided by another and the remainder is zero. For example, 15 divided by 3 is 5.

Factors are numbers that when combined in a multiplication equation give the product. The factor of a number is a whole number that divides it exactly. For example, 1, 2, 4, and 8 are factors of 8. A common factor of two or more numbers is a factor of all the numbers. The greatest common factor (GCF) is the greatest number in a list of common factors of the numbers.

A multiple is a number that is the product of a given number and a whole number. For example, the multiples of 3 are 3, 6, 9, 12, etc. The common multiple of two numbers is any number that is a multiple of both numbers. The least common multiple (LCM) of two or more numbers is the least number in the list of their common multiples.

It may be helpful to create a divisibility game. On small pieces of paper, write the numbers 1 to 100. Put the 100 pieces of paper in a bag or a hat. Have the student draw two pieces of paper from the hat. Help the student determine if one number is divisible by the other number. A strong knowledge of the multiplication tables will be helpful.

A similar game can be created for factors and multiples. On small pieces of paper, write the numbers 1 to 100. Put the 100 pieces of paper in a bag or hat. Have the student draw one piece of paper from the hat. Help the student determine the factors and multiples of the number.

It may be helpful to note that there are an infinite amount of multiples for a specific number, but there are only a certain amount of factors for the same number.

**Example 1:** Find the least common multiple (LCM) of 4 and 6.

Step 1: Determine the multiples of 4 and 6. (Multiply 4 times 1, 2, 3, 4, etc. and the same with the number 6)

 multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36,. . . .
 multiples of 6: 6, 12, 18, 24, 30, 36,. . . . . .

Step 2: List the common multiples.

common multiples of 4 and 6: 12, 24, 36,. . . .

Step 3: The least common multiple is the least number in the list of common multiples, which is the number 12.

Answer: The LCM of 4 and 6 is 12.

**Example 2:** Find the greatest common factor (GCF) of 24 and 56.

Step 1: Determine the factors of 24 and 56.

 factors of 24 are: 1, 2, 3, 4, 6, 8, 12, and 24
 factors of 56 are: 1, 2, 4, 7, 8, 14, 28, and 56

Step 2: List the common factors of 24 and 56.

Common factors of 24 and 56 are: 1, 2, 4, and 8

Step 3: The greatest common factor is the greatest value in the list, which is the number 8.

Answer: The GCF of 24 and 56 is 8.

**Compare Whole Number Equations - B**Comparing whole number equations involves determining which side of the equation is larger in value than the other. Knowledge of the greater than (>), less than (<), and equal to (=) signs is needed for this skill.

It may be helpful to review the ordering symbols with the student.



**Example 1:** 11,025 ÷ 105 ? 70 x 2

 (1) 11,025 ÷ 105 = 105
 (2) 70 x 2 = 140
 (3) 105 ? 140

Step 1: Simplify the expression on the left.
Step 2: Simplify the expression on the right.
Step 3: Rewrite the mathematical sentence with the new numbers and determine which symbol to place between the two numbers.

The answer is: 11,025 ÷ 105 < 70 x 2.

**Example 2:** 200,000 ÷ 20 ? 20,000 ÷ 2

 (1) 200,000 ÷ 20 = 10,000
 (2) 20,000 ÷ 2 = 10,000
 (3) 10,000 ? 10,000

Step 1: Simplify the expression on the left.
Step 2: Simplify the expression on the right.
Step 3: Rewrite the mathematical sentence with the new numbers and determine which symbol to place between the two numbers.

The answer is: 200,000 ÷ 20 = 20,000 ÷ 2.

**Multiple Operations: Whole Numbers - B**Multiplication skills at this level include operations with multi-digit numbers (237 x 56) which require regrouping (carrying, borrowing, renaming) when the product of the numbers in the ones or tens (hundreds, thousands, etc.) position are equal to or greater than ten. Necessary division skills include long division, remainders, and two-digit quotients.

When performing multiple operations on expressions, it is important to remember that operations inside parentheses are completed first. If there are two sets of parentheses, then the set that comes first when reading from left to right is completed first.

**Example 1:** (237 x 56) ÷ 12 = ?

 (1) 237 x 56 = 13,272
 (2) 13,272 ÷ 12 = ?
 (3) 13,272 ÷ 12 = 1,106

Step 1: Multiply the numbers inside the parentheses. (237 x 56)
Step 2: Rewrite the problem with the new value in place of the parentheses.
Step 3: Divide 13,272 by 12.

Answer: 1,106

**Whole Numbers: Story Problems**Story problems (word problems) require students to read passages, determine the question being asked, identify the elements needed to solve the problem, decide on the correct operation or operations (addition, subtraction, multiplication, division), and find a solution. The student's textbook may have word problems that he or she can use to practice these five steps. Remember, some story problems have extra information.

One of the best ways to understand story problems is to write them. Help the student come up with a story problem like the one below:

Sam went to the store with $81. At the store, he saw Michelle, who owed him $16. Michelle had $19.00 more than what she owed Sam. Michelle paid Sam what she owed him, and Sam bought a model car set for $31. How much money did Sam go home with?

The answer is $66.

**Subtract Decimals: Hundred Thousandths**Subtracting decimal numbers with more than one decimal position (columns of numbers) is very similar to subtracting whole numbers. Subtracting decimal numbers requires the ability to regroup (carry, borrow, or rename) when the number being subtracted is greater than the other number.

**Example:** 7.36925 - 4.8217 =


Step 1: Write the problem vertically. Remember to line up the decimal points and place a zero at the end of 4.8217 to hold the hundred thousandths place.
Step 2: Subtract the numbers in the hundred thousandths column (5 - 0 = 5). Place the 5 in the hundred thousandths place.
Step 3: Before the numbers in the ten thousandths column can be subtracted we must "borrow" or "trade" from the thousandths column. Cross out the 9 and make it an 8, then make the 2 in the ten thousandths column a 12. Now, subtract the numbers in the ten thousandths column (12 - 7 = 5). Place the 5 in the ten thousandths place.
Step 4: Subtract the numbers in the thousandths column (8 - 1 = 7). Place the 7 in the thousandths place.
Step 5: Subtract the numbers in the hundredths column (6 - 2 = 4). Place the 4 in the hundredths place.
Step 6: Before the numbers in the tenths column can be subtracted, we must "borrow" or "trade" from the ones column. Cross out the 7 and make it a 6, then make the 3 in the tenths column a 13. Now, subtract the numbers in the tenths column (13 - 8 = 5). Place the 5 in the tenths place.
Step 7: Subtract the numbers in the ones column (6 - 4 = 2). Place the 2 in the ones place.

Answer: 7.36925 - 4.8217 = **2.54755**

**Divide Decimals by Whole Number**Dividing a decimal number by a whole numberis very similar to dividing whole numbers. The decimals point must remain in the same position in the answer.

**Example**: Solve 18.9 divided by 9 = ?

Step 1: Write the problem in long division format.
Step 2: Division follows the same format as with whole numbers. 9 goes into 18 two times because 9 x 2 = 18. Place 2 in the ones position. Subtract 18 from 18 resulting in 0. Bring down the 9.
Step 3: Place the decimal point.
Step 4: 9 goes into 9 one time because 9 x 1 = 9. Place 1 in the tenths position. Subtract 9 from 9 resulting in 0.

The answer is 2.1.

**Story Problems: Decimals**Decimal story problems (word problems) require students to read passages, determine the question being asked, identify the elements needed to solve the problem, decide on the correct operation or operations (addition, subtraction, multiplication, division), and find a solution.

Decimal story problems are especially difficult because students must master addition, subtraction, multiplication, and division of decimals before story problems can be understood.

When the student has mastered decimal calculations, it will be helpful to begin to talk with him or her about the different ways decimals are used in real world scenarios: taxes, interest, measurements, money, etc. Next, encourage the student to write his or her own decimal story problems by using the following pattern: 1) Choose an operation: addition, subtraction, multiplication, or division. 2) Choose a decimal scenario: purchasing merchandise or measuring. For instance, if the student chooses addition of decimals in a purchasing scenario, he or she could write a decimal story like this one.

**Example**: Mary Ann spent $3.26 on two candy bars. Michelle bought a pack of gum for $.85 and a magazine for $2.95. How much money did Mary Ann and Michelle spend?


Step 1: Rewrite the problem vertically. Always line up the decimal points.
Step 2: Add the numbers in the hundredths position (6 + 5 + 5 = 16). Write the 6 in the hundredths position. Carry the 1 to the next column (tenths).
Step 3: Add the numbers in the tenths column, including the number carried over from the previous column (1 + 2 + 8 + 9 = 20). Write the 0 in the tenths position. Carry the 2 to the next column (ones).
Step 4: Bring the decimal point down.
Step 5: Add the numbers in the ones position, including the number carried over from the previous column (2 + 3 + 2 = 7). Write the 7 to the left of the decimal point. Bring down the dollar sign.

Answer: $3.26 + $0.85 + $2.95 = $7.06

**Fractions: Estimation - C**An estimate is an approximate calculation. In other words, an estimate is close to the actual answer, but not exact. When estimating fractions, it is important to understand how fractions work. The number on top of the fraction is called the numerator and the number on the bottom of the fraction is called the denominator.

When estimating a fraction, one of the most important concepts is the ability to determine whether a fraction is larger than or smaller than one-half, one-fourth, or three-fourths. This ability allows you to "round" a fraction to the closest "common" fraction -- 1/4, 1/2, 3/4, or 1.


**Example 1:**

To solve this problem, it may be helpful to draw the following pictures.


The first picture represents 5/12 of a pizza. The second picture represents 1/2 of a pizza. Since 5/12 of a pizza is close in value to 1/2 of a pizza, we can estimate that Charlie has 1/2 of a pizza.

We have estimated that Charlie has 1/2 of a pizza. Now we need to determine how much pizza each friend will get. To do this, we divide 1/2 by 2.

 
Step 1: Write the division problem out. Remember to place a denominator of 1 under the whole number 2 to make the whole number a fraction.
Step 2: To divide two fractions, the rule is "flip the second fraction and multiply." When we flip 2/1 we get 1/2. Now we multiply 1/2 by 1/2 to get 1/4. Remember, when multiplying fractions, multiply the numerators together then multiply the denominators together.


**Example 2:**

To solve this problem, it may be helpful to draw the following pictures.


The first picture represents 5/6 of a cup. Since 5/6 is closest to 3/4 cup, we can estimate that 5/6 is 3/4 cup. The second picture represents 2/9 and the third picture represents 1/4. Since 2/9 and 1/4 are close to the same amount, we can estimate that 2/9 is 1/4.

We have estimated that 5/6 is 3/4 and 2/9 is 1/4. To determine the approximate amount of raisins that Dehlia will need, we add 3/4 and 1/4. 3/4 + 1/4 = 1

Answer: Dehlia will need about 1 cup of raisins to make both recipes.

**Multiplying Integers**Integers are the set of positive and negative whole numbers, including zero. Students should understand how integers appear on a number line. Numbers to the right of 0 on a number line are positive and numbers to the left of 0 are negative. The number -3 is a negative integer and the number 3 is a positive integer. The integer 0 is neutral.

To multiply and divide integers, follow these rules.

 •The product of two positive integers is positive (Example: **9 x 4 = 36**).

 •The product of a positive integer and a negative integer is negative (Example: **9 x -4 = -36**).

 •The product of two negative integers is positive (Example: **-9 x -4 = 36**).

Notice from the above examples that when you multiply integers with the same sign, the answer is positive. When multiplying integers with different signs, the answer is negative.

The order of operations must be followed when working with grouping symbols and/or multiple-step operations. The order of operations are as follows:
 1. Work inside grouping symbols
 2. Multiply and divide from left to right
 3. Add and subtract from left to right

**Example 1:** (3 x -5) + 7 = ?

 (1) 3 x -5 = -15
 (2) -15 + 7 = -8

Step 1: Using the order of operations, calculate within grouping symbols. 3 x -5 = -15, so we can replace the parentheses with the value of -15.
Step 2: Perform the addition.

Answer: -8

**Example 2:** 8(-3 x 4) x (-5 + 7) = ?

 (1) -3 x 4 = -12 and -5 + 7 = 2
 (2) 8(-12) x 2 = ?
 (3) 8(-12) = -96
 (4) -96 x 2 = -192

Step 1: Work inside grouping symbols.
Step 2: Rewrite the problem with the new numbers.
Step 3: Multiply from left to right, 8 x -12 = -96
Step 4: Multiply -96 by 2 to get -192.

Answer is: -192

**Story Problems Integers**Integers are the set of positive and negative whole numbers, including zero. In integer story problems, students must determine the elements of the story that make up an integer equation, decipher the correct operation, and solve the problem.

To begin, the student should be familiar with how to add, subtract, multiply, and divide integers. Use the following definitions and examples to review adding and subtracting integers.

When adding two integers with the same sign, add their absolute values. Then give the sum (answer) the sign of the integers.

 -3 + -2 = ?
 |-3| + |-2| = ?
 3 +2 = 5, then make the result negative.

Answer: -5

When adding integers with different signs, first find their absolute values. Then subtract the lesser absolute value from the greater absolute value, and give the result the sign of the integer with the greater absolute value.

 -7 + 3 = ?
 |-7| = 7 and |3| = 3 (find the absolute values)
 7 - 3 = ? (subtract the lesser from the greater)
 7 - 3 = 4
 -7 + 3 = -4 (The result is given the sign of the greater integer.)

Subtracting integers is the same as adding the opposite.

 3 - -7 = ?
 3 + +7 = 10 (add the opposite)
 3 + 7 = 10

Use the following definitions and examples to review multiplying and dividing integers.

 •The product of two positive integers is positive. Example: **9 x 4 = 36**.

 •The product of a positive integer and a negative integer is negative. Example: **9 x -4 = -36**.

 •The product of two negative integers is positive. Example: **-9 x -4 = 36**.

 •The quotient of two positive integers is positive. Example: **9 ÷ 3 = 3**.

 •The quotient of a positive integer and a negative integer is negative. Example: **9 ÷ -3 = -3**.

 •The quotient of two negative integers is positive. Example: **-9 ÷ -3 = 3**.

Once the student is comfortable with these skills, help him or her apply these skills in a real life scenario.

**Example 1:** The temperature on Sunday was -9 degrees, and the temperature on Monday was -3 degrees. How much warmer was it on Monday than on Sunday?

 **(1) Sunday: -9 degrees
 Monday: -3 degrees
 (2) -3 - (-9) = ?
 (3) -3 + 9 = ?
 (4) -3 + 9 = 6**

Step 1: Identify the relevant information in the story problem.
Step 2: Determine the necessary operation. The story requires the difference to be found.
Step 3: Subtracting a negative number makes a positive.
Step 4: Complete the addition problem.

If the student has difficulty with the above step, draw a number line and label the points -9 and -3 on the number line. Count the difference.

Answer: Monday was 6 degrees warmer than Sunday.

The student should review finding averages with integers and be able to apply this skill to story problem situations.

**Example 2:** During a diving course for diving certification, John had to make 5 dives. His first dive was 20 feet down, his second was 30 feet down, his third was 55 feet down, his fourth was 75 feet down and his last was 100 feet below the surface. What was the average depth of all 5 dives?
Step 1: Identify the relevant information. Below the surface represents negative integers so -20, -30, -55, -75, -100
Step 2: Determine the necessary operations. Find the sum of the numbers and divide by the total amount of numbers.
Step 3: (-20) + (-30) + (-55) + (-75) + (-100) = -280
Step 4: -280÷ 5 = -56

Answer: 56 feet below the surface (because -56 represents 56 feet below the surface)

**Percent One Number is of Another**Percent means "per one hundred." When you state that 70% of the students ate tacos, then you are stating that 70 out of 100 students ate tacos. If 35 out of 50 students ate tacos, then 70% ate tacos because 35 out of 50 (35/50) is equal to 70 out of 100 (70/100). To understand percents, students must draw upon knowledge of decimal numbers, ratios, and fractions.

The following example should help the student better understand the steps needed for determining the percent one number is of another.
**Example:** What percent of 45 is 36?

 
Step 1: Rewrite the equation. Since the percent is what we want to find, we can represent it with the variable n%. Note that the word "of" means multiply and the word "is" means equal.
Step 2: Change n% to a fraction and multiply it by 45. n% times 45 should equal 36. Write the new equation.
Step 3: Multiply both sides of the equation by 100 to begin to isolate the n on one side of the equal sign.
Step 4: Rewrite the equation with the new numbers.
Step 5: Divide both sides of the equation by 45.
Step 6: 3600 ÷ 45 = 80.

Answer: 80%

**Story Problems Percents - B**Story problems, also called word problems, present addition, subtraction, multiplication, and division percentage problems in text format. Students must calculate discounts, interest, and sales tax.

A discount is an amount that is subtracted from an original amount.

**Example 1:** Helen receives a 20% discount of merchandise in the Hammer Hardware Store. The original price on a steel wrench is $8.00. How much will Helen have to pay for the wrench?

 (1) 20% of $8.00
 (2) $8.00 x (20%) = $8.00 x (0.20) = $1.60
 (3) $8.00 - $1.60 = $6.40

Step 1: Write out the important information from the problem. The problem is asking us to find 20% of $8.00 and then subtract that amount from $8.00.
Step 2: Multiply $8.00 by 20% to find the discount amount. Remember to convert the percent into a decimal before multiplying.
Step 3: Subtract the amount of the discount from the original price.

Answer: $6.40

The following formula is used to calculate simple interest:

 **Interest = Principal x Rate x Time**

"Principal" is the amount being borrowed.
"Rate" is the annual interest rate (given in the form of a percent).
"Time" refers to the amount of time the borrower has to pay back the loan.

**Example 2:** Sandy wants to borrow $3,000 to buy a new computer. He wants to borrow the money for 2 years at a simple interest rate of 9% per year. How much interest will Sandy pay over the two years?

 (1) $3,000 x 9% = $3,000 x 0.09 = $270
 (2) $270 x 2 = $540

Step 1: Multiply the amount Sandy is borrowing by the interest to find the interest per year. Remember to convert the percentage into a decimal before multiplying. Sandy will pay $270 per year interest.
Step 2: Multiply the yearly interest times the number of years.

Answer: $540

**Example 3:** A sweater costs $67.93. The sales tax in the state is 7.83%. What is the total cost of the sweater?

 (1) $67.93 x 7.83% = $67.93 x 0.0783 = 5.318919
 (2) 5.318919 ~ $5.32
 (3) $67.93 + $5.32 = $73.25

Step 1: Determine the amount of tax by multiplying $67.93 by 7.83%. Remember to convert 7.83% into a decimal before multiplying.
Step 2: Write 5.318919 as a money amount to determine the amount of tax.
Step 3: Add the original price and the amount of the tax to determine the total cost of the sweater.

Answer: $73.25

**Scientific Notation**Scientific notation is a condensed way to write very large or small numbers without including each digit. Scientific notation is a number written as the product of a number between 1 and 10 and a power of 10.

To write a large number using scientific notation, count the digits (from right to left) to be represented by a power of 10. 123,000,000 can be written in scientific notation as 1.23 x 10 to the 8th power. To write a small number, count the digits from left to right. To undo scientific notation, move the decimal point the same number of places as the exponent in the power of ten.

**Example 1:** 
Answer: 62,900,000 (move the decimal 7 places to the right)

**Example 2:** 
Answer: The missing exponent would be 4.

**Place Value: Whole Numbers - C**Each digit of a number is given a value for the position it is located. For example, in the number 56 the place value of the 6 is ones. The place value of the 5 is tens.

It may be helpful to develop a place value chart like the one below to help the student visualize place value.



**Example 1:** What does the digit 5 mean in 7,235,689?

Solution: Help the student fill in the place value chart with the correct digits (as seen in the place value chart below). Then use the chart to determine the meaning of the indicated number.



Since the 5 is in the thousands column, the 5 in 7,235,689 means **5 thousands**.

**Example 2:** In which place is the underlined number?

 123,458,079

Solution: Help the student fill in the place value chart with the correct digits. Then use the chart to determine the meaning of the indicated number.

The 3 is in the **millions** place.

**Rounding and Estimation - E**Rounding and estimation are used to express numbers to the nearest tenth, hundredth, thousandth and so forth. Real world applications use rounding and estimation to make numbers more manageable and understandable. For example, television producers often round large numbers so that reporters can state them in a simple manner. If the United States produced 134,995,659 ounces of gold, a reporter might state that, "The United States' gold production this year was 135,000,000 ounces."

An interesting method for improving the student's rounding and estimation skills is to create a list of numbers. Help the student round each number. Remember to look to the right of the place value you wish to round. Numbers less than 5 are rounded down, while numbers 5 or greater are rounded up.
EXAMPLES:

 **34 rounded to the nearest ten is 30.**
 **37 rounded to the nearest ten is 40.**

To help the student round decimal numbers, review decimal places with him or her. For example, in the number 6,879.342

 6 = thousands
 8 = hundreds
 7 = tens
 9 = ones
 3 = tenths
 4 = hundredths
 2 = thousandths

Once the student understands decimal places, ask him or her to round 6,879.342 to the nearest tenth. Look at the first digit to the right of the tenths place. The hundredths place is less than 5, so the tenths place remains unchanged. 6,879.342 rounded to the tenths place is 6,879.3

Estimation is also an important skill for simplifying and managing large numbers. For example, what is the estimated value of 37 x 11? Round each of the whole numbers to the tens place. 37 to 40 and 11 to 10. Think 40 x 10 = 400. Therefore, the estimated value of 37 x 11 is 400.

**Example 1:** Estimate the answer to the nearest whole number.
 729.99 + 66.072 = ?
Answer: Round 729.99 to 730 and round 66.072 to 66. 730 + 66 = 796

**Place Value: Decimals - B**In place value, the student must determine what a digit in a given number represents. In place value with decimals, the students must determine the place value for digits to the right of the decimal place.

It may be helpful to review the diagram below to help determine decimal place value.



In the number 32.4578, the 4 represents 4 tenths.



**Example 1:** What does the digit 8 mean in 57.929384?

The answer is hundred thousandths.

**Example 2:** In what place is the underlined digit?

 143.680259

Answer: The underlined digit is in the millionths place.

**Dividing Integers**Integers are the set of positive and negative whole numbers, including zero. To find the quotient (answer to a division problem) of two integers, the following rules apply:
The quotient of two integers with different signs is negative. Example: **16 ÷ -4 = -4**.
The quotient of two integers with the same sign is positive. Examples: **16 ÷ 4 = 4** and **-16 ÷ -4 = 4**.

Operations within parentheses are completed first. After performing operations within parentheses, perform all multiplication and division in order from left to right. The last step is to perform all addition and subtraction in order from left to right. (It may be helpful here to review order of operations and/or multiplying with integers.)

**Example 1:** 4(-3 x 2) ÷ (12 ÷ 2) = ?

 (1) -3 x 2 = -6 and 12 ÷ 2 = 6
 (2) 4(-6) ÷ 6 = ?
 (3) -24 ÷ 6 = ?
 (4) -24 ÷ 6 = -4

Step 1: Perform operations within parentheses: ( -3 x 2 = -6) and (12 ÷ 2 = 6).
Step 2: Write out the problem, replacing the values within the parentheses with the new values.
Step 3: Perform multiplication or division in order from left to right. Multiply first because it comes first when reading from left to right. 4(-6) = -24.
 Step 4: Divide -24 by 6 to get -4. Remember the quotient of two integers with different signs is negative.

Answer: -4

The following example illustrates the use of rules for dividing integers using "is greater than" (>) and "is less than" (<).

**Example 2:** -24 ? 3(10 ÷ -2)

 (1) 10 ÷ -2 = -5
 (2) -24 ? 3(-5)
 (3) 3 x -5 = -15
 (4) -24 ? -15
 (5) -24 < -15

Step 1: Perform operations within parentheses. 10 ÷ -2 = -5.
Step 2: Rewrite the problem with -5 in place of the parentheses.
Step 3: Multiply 3 x -5 to get -15.
Step 4: Rewrite the problem with -15 in place of 3(-5).
Step 5: To determine which symbol to place between -24 and -15, think of the integers as being money. -24 would be like owing someone $24 and -15 would be like owing someone $15. Since owing $24 is more in debt than owing $15, -24 is less than -15.

Answer: -24 < -15

**Integers: Multiple-step Computation**Integers are positive and negative whole numbers, including zero.

Before computing with integers, let's first review the rules of operations on integers.

When adding two integers with the same sign, add their absolute values. Then give the sum (answer) the sign of the integers.

 -3 + -2 = ?
 |-3| + |-2| = ?
 3 +2 = 5, then make the result negative.

Answer: -5

When adding integers with different signs, first find their absolute values. Then subtract the lesser absolute value from the greater absolute value, and give the result the sign of the integer with the greater absolute value.

 -7 + 3 = ?
 |-7| = 7 and |3| = 3 (find the absolute values)
 7 - 3 = ? (subtract the lesser from the greater)
 7 - 3 = 4
 -7 + 3 = -4 (The result is given the sign of the greater integer.)

Subtracting integers is the same as adding the opposite.

 3 - -7 = ?
 3 + +7 = 10 (add the opposite)
 3 + 7 = 10

When multiplying integers, the product (answer) of two integers with the same sign is positive. The product of two integers with different signs is negative.

 (-5)(-6) = 30
 (-5)(6) = -30

When dividing integers, the quotient (answer) of two integers with the same sign is positive. The quotient of two integers with different signs is negative.

 -6 ÷ -3 = 2
 -6 ÷ 3 = -2

When performing computations with more than one operation, follow the rules for the order of operations.The order of operations is as follows:
 1. Perform operations within parentheses, braces, or brackets.
 2. Multiply and divide from left to right
 3. Add and subtract from left to right

**Example 1:** (-2 - 5) + 10 = ?

 (1) -2 - 5 = -7
 (2) -7 + 10 = ?
 (3) -7 + 10 = 3

Step 1: Perform the operation within parentheses, -2 - 5 = -7.
Step 2: Replace the -7 for the value of the parentheses.
Step 3: Complete the addition problem.

Answer: 3

When calculating problems with <, >, and =, perform the calculations on each side to determine the values of each side.

**Example 2:** -(57 - -10) ? (-12 - -22)

 (1) -(57 - -10) = -(57 + 10) = -(67) = -67
 (2) -12 - -22 = -12 + 22 = 10
 (3) -67 ? 10
 (4) -67 < 10

Step 1: Evaluate the value in the first set of parentheses. Remember, subtracting a negative is the same as adding the opposite. Then distribute the negative sign to the answer.
Step 2: Evaluate the value in the second set of parentheses.
Step 3: Rewrite the mathematical sentence with the new values.
Step 4: Negative 67 is less than positive 10.

Answer: <