Study Guide

7th Grade Algebra Practice
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**Conversion: Variable Expressions/Words**Learning to convert variable expressions into words or words into variable expressions is an important problem solving skill. In this tutorial, actual solutions to the problems will not be determined.

It is important for the student to understand the difference between an expression and an equation.

Expressions are variables or combinations of variables, numbers, and symbols that represent a mathematical relationship. Expressions do not have equal signs, but can be evaluated or simplified.
 Example: *y* - 6

Equations are expressions that contain equal signs. They can be solved, but not evaluated.
 Example: *n* + 5 = 9

**Writing Variable Expressions To Represent Word Phrases:**
To represent a word phrase or story problem using algebraic symbols, there are three steps that the student should follow.
Step 1: Choose a variable to represent the unknown quantity, or use the variable provided. Some problems will tell the student which variable to use.
Step 2: Look for key words in the phrase or story problem that indicate the use of a particular operation. The chart below shows several key words with their corresponding operations.

 
Step 3: Use the chosen variable, relevant information, and operation key words to set up the variable expression.

**Example 1**:
**Translate the following story problem into an expression.**

In August, a realtor sold 6 less than 4 times the amount of homes she sold in January. How many homes did the realtor sell in August? Let *h* = the number of homes sold in January.

 (1) *h* = the number of homes sold in January
 (2) The key words are *less than* and *times*.
 (3)

Step 1: Identify the variable.
Step 2: Look for key words and use the chart above to determine the necessary operations. *Less than* implies subtraction and *times* implies multiplication.
Step 3: Set up the variable expression. Be sure to double check that all parts of the word phrase have been represented. Also, it is important to recognize that subtraction, although usually stated first in the word phrase, is normally placed at the end of the expression. This can be tricky for many students since they are taught to read from left to right.

**Answer:** 4*h* - 6

**Example 2:**
**Translate the following word phrase into an expression.**

nine less than ten times the sum of a number, *y*, and three

 (1) *y* = the number
 (2) The key words are *less than*, *times*, and *sum of*.
 (3)
Step 1: Identify the variable.
Step 2: Look for key words and use the chart above to determine the necessary operations. *Less than* implies subtraction, *times* implies multiplication, and *sum of* implies addition (usually as a quantity written in parentheses).
Step 3: Set up the variable expression. Be sure to double check that all parts of the word phrase have been represented. Also, it is important to recognize that subtraction, although usually stated first in the word phrase, is normally placed at the end of the expression.

**Writing Word Phrases To Represent Variable Expressions:**
A similar process is used to write word phrases from variable expressions. In most cases, the student will be provided with a choice of word phrases as opposed to actually having to create one. This is because a variety of different phrases could accurately represent one expression. The student has two options.
Option 1: The student can review the answer choices and apply the same three steps listed above to determine the corresponding variable expression.
Option 2: The student can review the variable expression determining the operations used, values, and order in which it is set up. This information will guide the student to the correct word phrase.

**Example 3:** Translate the following expression into words.
 
**(A)** the difference of half the peanuts, *p*, and four
**(B)** four plus twice the number of peanuts, *p*
**(C)** the sum of one-half the number of peanuts, *p*, and four
**(D)** one-half the sum of the number of peanuts, *p*, and four

**Solution:**
The student should rule out option **(A)** because it mentions *difference* which implies subtraction. This description would represent the expression ?(*p*) - 4.
The student should rule out option **(B)** because it mentions *twice* which implies multiplying by 2 instead of ?. This description would represent the expression 2*p* + 4.
The student should rule out option **(D)** because it represents taking one half of the sum, which means you have to add the number, *p*, and four first, and then multiply by one-half. This description would represent the expression ?(*p* + 4).

**Answer:** The correct answer is option **(C)**.

**Expressions: Addition**Expressions are number sentences which do not have equal signs, but need to be evaluated or simplified.
 Example: y - 6

An equation is a number sentence that does have an equal sign.
 Example: y - 6 = 14

**Example 1:** Evaluate the expression x + 23, when x = 5.

 5 + 23

Solution: Substitute the value 5 in place of x in the expression.

Answer: 28

**Example 2:** For x = -7, find 2x + -12.

 (1) 2(-7) + -12
 (2) -14 + -12
 (3) -26

Step 1: Substitute -7 in for the value of x.
Step 2: Multiply 2 by -7 and rewrite the expression with the new value.
Step 3: Add -14 and -12.

Answer: -26

**Example 3:** Write a mathematical expression to represent the following:
 The sum of a number and 23.

Solution: Remember that "sum" is the answer to an addition problem, so the expression is x + 23.

Answer: x + 23

**Expressions: Multiplication**Expressions are number sentences which do not have equal signs, but need to be evaluated or simplified.
 Example: y - 6

An equation is a number sentence that does have an equal sign.

 Example: y - 6 = 14

**Example 1:** Evaluate the expression below for x = 18.

 5 + 3x

 (1) 5 + 3(18)
 (2) 5 + 54
 (3) 59

Step 1: Replace x with 18.
Step 2: Multiply 3 by 18 to get 54.
Step 3: Add 5 and 54.

Answer: 59

**Example 2:** Write a mathematical expression that represents the following word expression.

 four times a number less 6

 (1) four times x less 6
 (2) 4x less 6
 (3) 4x - 6

Step 1: Replace the words "a number" with a variable (x was chosen)
Step 2: "four times x" can be written as 4x. Make this replacement.
Step 3: "less 6" can be written as "- 6." Make this replacement and the expression is complete.

Answer: 4x - 6

**Expressions: Story Problems**Story problems for equations (word problems) require students to read passages, determine variables, write equations, and solve.

Expressions are number sentences which do not have equal signs, but need to be evaluated or simplified.
 Example: y - 6

Equations are number sentences which contain equal signs.
 Eample: n + 5 = 9

**Example:** The length of John's model airplane is 10 inches more than twice the width. The width is 15 inches. What is the length of John's model airplane?

 (1) L = 2W + 10
 (2) L = 2(15) + 10
 (3) L = 30 + 10
 (4) L = 40

Step 1: The length is equal to two times the width plus 10. Develop a formula with W = Width and L = Length.
Step 2: Replace W with 15.
Step 3: Calculate the right side of the equation by adding 30 and 10.

Answer: The length of John's model airplane is 40 inches.

**Distance/Rate/Time**The formula for solving distance (D), rate (R), and time (T) is:
 D = R x T
Students should be able to read story problems, decipher two elements of the distance formula, plug the elements in the distance formula, and solve.

The following are examples of problems requiring the D = R x T formula.

**Example 1:** If Carson drove 55 miles per hour on the freeway for 495 miles, how long did he drive?

 
Step 1: Determine the distance, rate, and time values and substitute them into the D = R x T formula. D = 495, R = 55, and T = ?
Step 2: Solve for T (Time) by dividing both sides of the equation by 55.

Answer: Carson drove for 9 hours.

**Example 2:** Stan ran for 164.5 hours. He ran a total of 2500 miles. What was the rate of speed that Stan ran? (Round miles per hour to the nearest hundredth).

 
Step 1: Determine the distance, rate, and time values and substitute them into the D = R x T formula. D = 2500, R = ? and T = 164.5.
Step 2: Solve for R (Rate) by dividing both sides of the equation by 164.5.
Step 3: Round the answer to the nearest hundredth.

Answer: Stan ran at a rate of 15.20 miles per hour.

**Exponential Notation - C**An exponent is a number that represents how many times the base is used as a factor. The base number 5 to the 3rd power translates to 5 x 5 x 5 which equals 125. 5 to the 3rd power is not 5 x 3. To perform operations with exponents, all exponential properties must be understood.

Have the student find the equivalent whole number forms of these exponential numbers:

 
**Scientific Notation:**

Scientific notation is based upon exponential properties and is used to communicate very large or very small numbers. Scientific notation deals with significant digits. The most significant digit in a number is the first non-zero digit in the number (reading from left to right). To write a number using scientific notation, place the decimal point to the right of the most significant digit and count the digits between the new placement of the decimal point and the old placement of the decimal point. The number of places that the decimal point moved will be represented by a power of 10.

**Example 1:** Write 123,000,000 using scientific notation.

 
Step 1: Determine where the decimal point is in the number to be written using scientific notation.
Step 2: Place the decimal point to the right of the most significant digit and count the number of places the decimal point was moved.
Step 3: Write all of the significant digits (with the decimal in the new postion) and multiply by 10 to a power. The power on the ten is the number of places that the decimal point was moved. The power is positive because the decimal point was moved from the right to the left.

Answer: 

To take a number out of scientific notation, move the decimal point the same number of places as the exponent in the power of ten.

**Example 2:** 
Answer: 62,900,000 (move the decimal 7 places to the right)

**Example 3:** Find the equivalent form.
 

Step 1: Write the number in the first set of parentheses in standard form and rewrite the expression.
Step 2: Write the number in the second set of parentheses in standard form and rewrite the expression.
Step 3: Add 6,000 and 200 to get 6,200.

Answer: 6,200

**Radicals and Roots**Mastering roots and radicals is an essential step toward learning advanced mathematics concepts. A radical sign looks like a check mark with a line attached to the top. The radical sign is used to communicate square roots.

The following rules are required to perform operations with roots and radicals.

1. If x multiplied by x equals y, then x is a square root of y. For example, 6 multiplied by 6 is 36, so 6 is a square root of 36. In fact, 36 is called a perfect square because its square root, 6, is a whole number. Most algebra text books contain a table of perfect squares.

2. -3 and 3 are both square roots of 9 because -3 x -3 = 9 and 3 x 3 = 9. 3 is referred to as the principal square root because it is the positive square root of 9.

3. To find the simplest radical form of a radical expression, factor the number under the radical sign (the radicand). The square root of 45 could be factored to be the square root of 9 multiplied by the square root of 5. The square root of 9 multiplied by the square root of 5 can be simplified further by finding the square root of 9. The result is 3 (the square root of 9) multiplied by the square root of 5.

 
**Example 1:** Find the equivalent form.

 
Solution: Multiply the numbers under the radical symbols.

 

**Example 2:** What symbol would best replace the ? in the given statement?
 
 A. =
 B. <
 C. >

There are two methods that can be used to solve this problem. Each method is shown and explained below.

Solution Method 1:

 
Step 1: Simplify the square root of 12 by making it the square root of 6 x 2.
Step 2: Further simplify the square root of 6 x 2 by making it the square root of 3 x 2 x 2. (If you multiply 3 x 2 x 2, you will get 12.)
Step 3: The square root of 3 x 2 x 2 becomes 2 times the square root of 3, because the square root of 2 x 2 is 2 and the 3 must remain under the square root symbol.
Step 4: Two times the square root of 3 can also be written as the square root of 3 plus the square root of 3.
Step 5: Now, we can make a comparison. We know that the larger a number is, the larger that number's square root will be. We can determine that the square root of 5 plus the square root of 7 will be greater than the square root of 3 plus the square root of 3 because 5 and 7 are both larger than 3.

Answer: C

Solution Method 2:

 
Step 1: Estimate the square root of 5, the square root of 7, and the square root of 12. This estimation can be done using a calculator.
Step 2: Add together the 2.24 and the 2.65 to get 4.89.
Step 3: Replace the question mark with the > symbol because 4.89 is greater than 3.46.

Answer: C

**Example 3:** Solve for the value of x.
 
 
Step 1: The square root of 36 is 6 because 6 x 6 = 36.
Step 2: Subtract 6 from each side of the equation to isolate the square root of x.
Step 3: The square root of x is equal to 8. We can replace the x with 64, since the square root of 64 is 8.
Step 4: Since the square root of 64 equals 8, the value of x is 64.

Answer: x = 64

**Properties - D**Students must be able to solve for a missing value in a given equation. Understanding properties such as the order of operations is the key to correctly solving these problems.

Please review the following rules with the student:

1. Multiplication by 0: the product of any integer and 0 equals 0.

 -3 x 0 = 0 3 x 0 = 0

2. Associative Property of Addition: (a + b) + c = a + (b + c).

 (1 + 2) + 3 = 1 + (2 + 3)

3. Associative Property of Multiplication: (a x b) x c = a x (b x c).

 (1 x 2) x 3 = 1 x (2 x 3)

4. Reciprocals: two numbers are reciprocals if their product equals 1.


5. Commutative Property of Addition: a + b = b + a

 1 + 2 = 2 + 1

6. Commutative Property of Multiplication: a x b = b x a

 1 x 2 = 2 x 1

7. Order of Operations:
A. When calculations for a given expression or equation require both addition and multiplication, the rule is to multiply first and add second.

 (3)(2) + 3 = ?
 6 + 3 = ?
 6 + 3 = 9

B. The number outside the parentheses is multiplied with each number within the parentheses:
x(y + z) = xy + xz.

 (1) 3(x + y)
 (2) 3(x) + 3(y)
 (3) 3x + 3y

C. If a given expression contains both parentheses and brackets, calculations should be completed working from the innermost parentheses or bracket outward.

 
The following are sample questions using the above properties.

**Example 1:** Which answer best completes the number sentence?
 5 = (5 x 4) + (5 x 6)

 A. x (4 + 6)
 B. + (4 + 6)
 C. x (20 + 30)
 D. + (4 x 6)

Answer: A (because of rule 7B)

**Example 2:** Which one of the following best completes the number sentence?
 (2.3 + 3.1) x 5.6 = ?

 A. 3.5 x 5.6
 B. 7.2 x 5.6
 C. 9.1 x 5.6
 D. 5.4 x 5.6

Answer: D (because of rule 7A)

**Example 3:** What is the value of n in the following statement?
 13 x (3.4 x 0) = n

 A. 44.2
 B. 0
 C. 13
 D. 3.4

Answer: B (because of rule 1)

**Inequalities - A**An inequality is a number sentence that uses "is greater than", "is less than", or "is not equal to" symbols. For example, 6n > 4 is a number sentence with an inequality symbol.

It may be useful to review the inequality symbols.

 
**Example 1:** Solve for y: 8y > 40

 
Get the variable being solved for (y) on one side of the inequality and the whole number on the other. To do this, divide both sides by 8.

The correct answer is that y is greater than 5.

Inequalities can be represented as a value on a number line. The following number line represents the inequality 
 
**Example 2:** Which inequality represents the value shown on the number line below?

 
 A. n < 3
 B. n > 3
 C. n = 3

The answer is A. n < 3 because the dot on the number line is open.

**Multiple-step Story Problems - E**These problems are designed to test a student's ability to interpret data from story (word) problems. Answers are found by solving equations with multiple operations.

It may be helpful to develop a series of multiple-step word problems that relate to the student's activities, such as allowance. The following is a step-by-step example of a multiple-step story problem.

**Example 1:** On Saturday, Stella earned $3.50 for each hour of work. She earned $3.25 for each hour of work on Sunday. She worked 5 hours each day. How much money did she earn for both days?

 (1) $3.50 x 5 = ?
 $3.25 x 5 = ?
 (2) $3.50 x 5 = $17.50
 $3.25 x 5 = $16.25
 (3) $17.50 + $16.25 = $33.75

Step 1: Develop 2 separate equations. One to find the earnings on Saturday, and one to find the earnings on Sunday.
Step 2: Find the products of the two equations.
Step 3: Add the two products together.

Answer: Stella earned $33.75.

**Example 2:** Saman ate 3 times as many cookies as Alli. Alli ate 5 cookies less than Josh. Josh ate 10 cookies. How many cookies did Saman eat?

 (1) 10 - 5 = 5
 (2) 5 x 3 = 15

Step 1: Since Alli ate 5 cookies less than Josh, subtract 5 from 10 to determine the number of cookies she ate.
Step 2: Now that we know how many cookies Alli ate, we can determine the number of cookies Saman ate by multiplying 5 by 3.

Answer: Saman ate 15 cookies.

**Irrelevant/Missing Info - A**Students are presented with story problems that test their ability to read word problems, identify and utilize information pertinent to the question being asked, and disregard irrelevant information.

The following is an example of a story problem that contains irrelevant information.

**Example:** Cameron is twice as old as Gerardo. Chris is half Gerardo's age. Jelena, Cameron's 25 year old sister, is 7 years older than Cameron. Based on this information, how old is Gerardo?

Solution: Make a list of the ages.

 1. Cameron's age = 2 x Gerardo's age
 2. Chris's age = 1/2 x Gerarado's age
 3. Jelena's age = Cameron's age + 7

The question requires Gerardo's age to be found. The only age we are given is Jelena's, 25. We can determine that Cameron is 18 because we know that he is 7 years younger than Jelena. Because we know that Cameron is 18, we know that Gerardo is 9 since twice Gerardo's age equals Cameron's age. Therefore, Gerardo is 9 years old. Chris's age was the irrelevant piece of information.

Many word problems contain irrelevant information. Help the student practice breaking down information the story problems in his or her math textbook. As he or she learns this skill, he or she will become better at solving word problems.